

9 MAP PROJECTIONS AND REFERENCE SYSTEMS

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9.1 Introduction

A map is a projection of data usually from the real Earth, celestial body or imagined world to a plane representation on a piece of paper or on a digital display such as a computer monitor. Usually, maps are created by transforming data from the real world to a spherical or ellipsoidal surface (the generating globe) and then to a plane. The characteristics of this generating globe are that angles, distances or surfaces measured on it are proportional to those measured on the real Earth. The transformation from the curved surface into a plane is known as map projection and can take a variety of forms, all of which involve distortion of areas, angles, and/or distances. The types of distortion can be controlled to preserve specific characteristics, but map projections must distort other characteristics of the object represented. The main problem in cartography is that it is not possible to project/transform a spherical or ellipsoidal surface into a plane without distortions. Only a spherical or ellipsoidal shaped globe can portray all round Earth or celestial body characteristics in their true perspective.

The process of map projection is accomplished in three specific steps:

- 1) approximating the size and shape of the object (e.g., Earth), by a mathematical figure that is by a sphere or an ellipsoid;
- 2) reducing the scale of the mathematical representation to a generating globe (a reduced model of the Earth from which map projections are made) with the principal

scale or nominal scale that is the ratio of the radius of the generating globe to the radius of the mathematical figure representing the object [Earth] equivalent to the scale of the plane map; and
 3) transforming the generating globe into the map using a map projection (Figure 9.1).

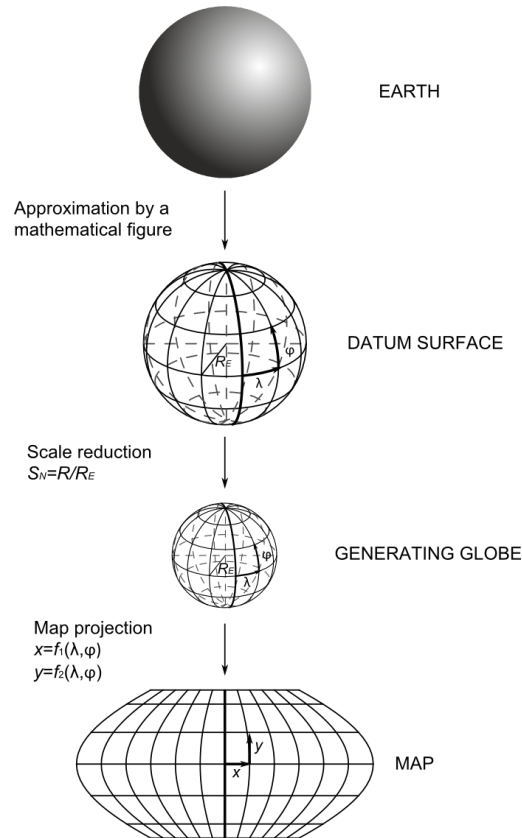


Figure 9.1. Map projection from the Earth through a generating globe to the final map (After Canters, 2002).

Map projections depend first on an assumption of specific parameters of the object (Earth) itself, such as spherical or ellipsoidal shape, radius of the sphere (or lengths of the semi-major and semi-minor axes of the ellipsoid), and a specific datum or starting point for a coordinate system representation. These assumptions form the basis of the science of Geodesy and are currently accomplished using satellite measurements usually from the Global Positioning System (GPS), Glonass, or Galileo (see section 9.2). Once these measurements are accepted, an ellipsoidal representation of coordinates is generated as latitude and longitude coordinates. Those coordinates can then be transformed through map projection equations to a plane Cartesian system of x and y coordinates. The general equations of this transformations have the following form:

$$x = f_1(\phi, \lambda), \quad y = f_2(\phi, \lambda)$$

where

- x is the plane coordinate in the east-west direction
- y is the plane coordinate in the north-south direction
- ϕ is the latitude coordinate
- λ is the longitude coordinate

The form of the functions f_1 and f_2 determines the exact transformation and the characteristics of the ellipsoidal or spherical representation that will be preserved.

Before addressing the specific types of transformations and the characteristics preserved, it is necessary to understand the geodetic characteristics of the ellipsoidal coordinates and how these are generated with modern satellite positioning systems.

9.2 Geodesy and Global Navigation Satellite Systems (GNSS)

Map projections have their largest and most frequent application in producing maps showing a smaller or bigger part of the Earth's surface. In order to produce the map of a region, it is necessary to make a geodetic survey of that region and then to visualise the results of such a survey. *Geodesy* is a technology and science dealing with the survey and representation of the Earth's surface, the determination of the Earth's shape and dimensions and its gravity field. Geodesy can be divided into applied, physical, and satellite geodesy .

Applied geodesy is a part of geodesy encompassing land surveying, engineering geodesy and management of geospatial information. Land surveying is a technique for assessing the relative position of objects on the Earth surface, when the Earth's curvature is not taken into account. Engineering geodesy is a part of geodesy dealing with designing, measuring, and supervising of constructions and other objects (*e.g.*, roads, tunnels and bridges).

Physical geodesy is a part of geodesy dealing with the Earth's gravity field and its implication on geodetic measurements. The main goal of physical geodesy is the determination of the dimensions of the geoid, a level surface modelling Earth, where the potential of the gravity field is constant. *Geometrical geodesy* is concerned with determination of the Earth's shape, size, and precise location of its parts, including accounting for the Earth's curvature.

Satellite geodesy is part of geodesy where satellites are used for measurements. In the past, exact positions of isolated spots on the Earth were determined in

astronomical geodesy, that is, by taking measurements on the stars. Measuring techniques in satellite geodesy are geodetic usage of Global Navigation Satellite Systems (GNSS) such as GPS, Glonass and Galileo.

A *satellite navigation system* is a system of satellites that provides autonomous geospatial positioning with global coverage. It allows small electronic receivers to determine their location (longitude, latitude, and altitude) to within a few metres using time signals transmitted along a line-of-sight by radio from satellites. Receivers calculate the precise time as well as position. A satellite navigation system with global coverage may be termed a global navigation satellite system or GNSS. As of April 2013, only the United States NAVSTAR Global Positioning System (GPS) and the Russian GLONASS are global operational GNSSs. China is in the process of expanding its regional Beidou navigation system into a GNSS by 2020. The European Union's Galileo positioning system is a GNSS in initial deployment phase, scheduled to be fully operational by 2020 at the earliest. France, India and Japan are in the process of developing regional navigation systems. Global coverage for each system is generally achieved by a satellite constellation of 20–30 medium Earth orbit satellites spread among several orbital planes. The actual systems vary but use orbital inclinations of $>50^\circ$ and orbital periods of roughly twelve hours at an altitude of about 20,000 kilometres.

Photogrammetry is an important technology for acquiring reliable quantitative information on physical objects and the environment by using recording, measurements and interpretation of photographs and scenes of electromagnetic radiation by using sensor systems. *Remote sensing* is a method of collecting and interpreting data of objects from a distance. The method is characterized by the fact that the measuring device is

not in contact with the object to be surveyed. Its most frequent application is from aerial or space platforms.

The study of the transformation from the Earth's surface model or generating globe to a two-dimensional representation requires the use of the following concepts: ellipsoid, datum, and coordinate system. Each of these is discussed below.

The *Earth's ellipsoid* is any ellipsoid approximating the Earth's figure. Generally, an ellipsoid has three different axes, but in geodesy and cartography, it is most often a rotational ellipsoid with small flattening (Figure 9.2).

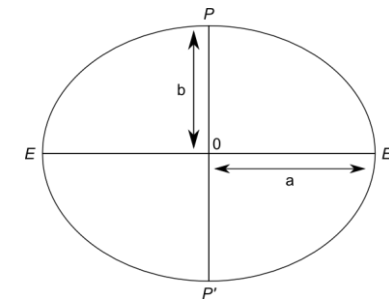


Figure 9.2. Terminology for rotational ellipsoid: EE' is the major axis, PP' is the minor axis and the axis of rotation, where a , is the semi-major axis and b is the semi-minor axis.

The rotational ellipsoid is a surface resulting from rotating an ellipse around a straight line passing through the endpoints of the ellipse. It is used to model the Earth. Famous Earth ellipsoids include the ones elaborated by Bessel (1841), and the more recently, WGS84 and GRS80 ellipsoids. Flattening is a parameter used to determine the difference between the ellipsoid and the

sphere. It is defined by the equation $f = \frac{a-b}{a}$, where

a and b are the semi-major and semi-minor axes, respectively. The semi-major axis a , is the Equatorial radius because the Equator is a circle. The semi-minor axis b is not a radius, because any planar section of the ellipsoid having poles P and P' as common points is an ellipse and not a circle.

Generally speaking, a datum is a set of basic parameters which are references to define other parameters. A *geodetic datum* describes the relation of origin and orientation of axes on a coordinate system in relation to Earth. At least eight parameters are needed to define a global datum: three for determination of the origin, three for the determination of the coordinate system orientation and two for determination of the geodetic ellipsoid. A two-dimensional datum is a reference for defining two-dimensional coordinates on a surface. The surface can be an ellipsoid, a sphere or even a plane when the region of interest is relatively small. A one-dimensional datum or vertical datum is a basis for definition of heights and usually in some relation to mean sea level.

The WGS84 and GRS80 ellipsoids were established by satellite positioning techniques. They are referenced to the centre mass of the Earth (*i.e.*, geocentric) and provide a reasonable fit to the entire Earth. The WGS84 datum provides the basis of coordinates collected from the GPS, although modern receivers transform the coordinates into almost any user selected reference datum.

The need for datum transformation arises when the data belongs to one datum, and there is a need to get them in another one (*e.g.*, WGS84 to North American Datum of

1927 or vice versa). There are several different ways of datum transformation, and readers should consult the appropriate geodetic references (see Further Reading section) or their device handbook.

9.3 Three-Dimensional Coordinate Reference Systems

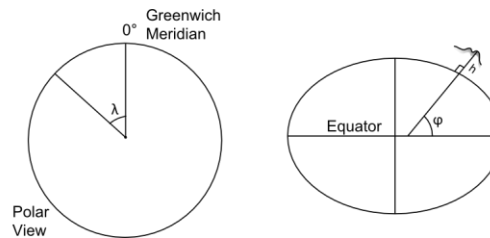


Figure 9.3. Geodetic or ellipsoidal coordinate system.

Geodetic coordinates are geodetic latitude and geodetic longitude, with or without height. They are also referred to as ellipsoidal coordinates.

Geodetic latitude is a parameter which determines the position of parallels on the Earth's ellipsoid and is defined by the angle from the equatorial plane to the normal one (or line perpendicular) to the ellipsoid at a given point. It is usually from the interval $[-90^\circ, 90^\circ]$ and is marked with Greek letter ϕ . An increase in geodetic latitude marks the direction of North, while its decrease marks the direction South. *Geodetic longitude* is a parameter which determines the position of the meridian on the Earth's ellipsoid and is defined by the angle from the prime meridian (that is the meridian of the Greenwich observatory near London) plane to the given point on the meridian plane. It is most often from the interval $[-180^\circ, 180^\circ]$ and is marked with Greek letter λ . An increase in geodetic longitudes determines the

direction of East, while a decrease determines the direction of West (Figure 9.3).

A geodetic datum should define the relation of geodetic coordinates to the Earth. Geodetic coordinates ϕ , λ and height h may be transformed to an Earth-centred, Cartesian three-dimensional system using the following equations:

$$X = (N + h) \cos \phi \cos \lambda$$

$$Y = (N + h) \cos \phi \sin \lambda$$

$$Z = (N(1 - e^2) + h) \sin \phi$$

where

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}, e^2 = \frac{a^2 - b^2}{a^2}.$$

If we wish to represent a large part of the Earth, a continent or even the whole world, the flattening of the Earth can be neglected. In that case, we speak about a geographic coordinate system instead of a geodetic coordinate system. Geographic coordinates are geographic latitude and geographic longitude, with or without height. They are also referred to as spherical coordinates. *Geographic latitude* is a parameter which determines the position of parallels on the Earth's sphere and is defined by the angle from the equatorial plane to the normal on the sphere at a given point. It is usually from the interval $[-90^\circ, 90^\circ]$ and is marked with Greek letter ϕ . An increase in geographic latitude marks the direction of North, while its decrease marks the direction South. *Geographic longitude* is a parameter which determines the position of the meridian on the Earth's sphere and is defined by the angle from the prime meridian plane to the given point on the meridian

plane. It is most often from the interval $[-180^\circ, 180^\circ]$ and is marked with Greek letter λ . An increase in geographic longitudes determines the direction of East, while a decrease determines the direction of West (Figure 9.4).

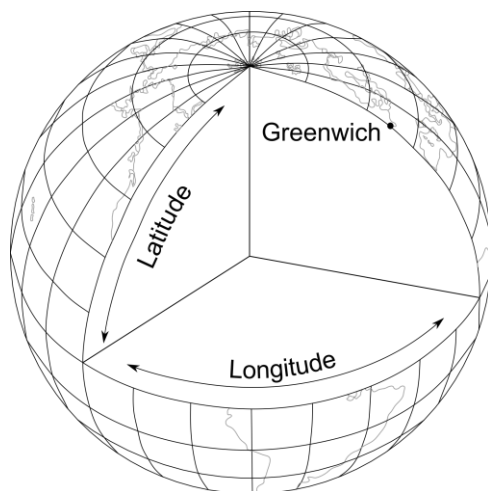


Figure 9.4. Geographic or spherical coordinate system: geographic latitude ϕ , geographic longitude λ .

Geographic coordinates ϕ , λ and height $h=0$ may be transformed to an Earth-centred, Cartesian three-dimensional system using the following equations:

$$\begin{aligned} X &= R \cos \phi \cos \lambda \\ Y &= R \cos \phi \sin \lambda \\ Z &= R \sin \phi \end{aligned}$$

where R is a radius of the spherical Earth.

A spherical coordinate system can be obtained as a special case of an ellipsoidal coordinate system taking

into account that flattening equals zero, $f = 0$, or equivalently stating that the second eccentricity equals zero, $e = 0$.

Sometimes, in geodetic and cartographic practice, it is necessary to transform Cartesian three-dimensional coordinates to spherical or even ellipsoidal coordinates. Furthermore, sometimes there is a need to make a transformation from one three-dimensional coordinate system to another one. The appropriate methods or equations exist, but the reader should consult the available literature (see Further Reading chapter).

9.4 Two-Dimensional Coordinate Reference Systems

Generally, for use of geospatial data, a common frame of reference is needed and this is usually done in a plane reference system. Because maps reside in a plane geometric system, the spherical or ellipsoidal coordinates, generated from satellite positioning systems or from any other surveying device, must be mathematically transformed to the plane geometry system. The simplest transformation is to assume that the plane x coordinate is equivalent to ϕ , and the plane y coordinate is equivalent to λ . The result is known as the Plate Carrée projection and although it is simple, it involves significant distortion of the coordinate positions and thus presents areas, most distances, and angles that are distorted or deformed in the plane.

More sophisticated transformations allow preservation of accurate representations of area or distance or angles, or other characteristics, but not all can be preserved in the same transformation. In fact, usually only a single characteristic, for example preservation of accurate representation of area, can be maintained, resulting in

distortion of the other characteristics. Thus, many different map projections have been developed to allow preservation of the specific characteristics a map user may require. The following sections provide discussion and the mathematical basis for transformations that preserve specific Earth characteristics, specifically area, angles, and distances.

The Universal Transverse Mercator (UTM) coordinate system is based on projections of six-degree zones of longitude, 80° S to 84° N latitude and the scale factor 0.9996 is specified for the central meridian for each UTM zone yielding a maximum error of 1 part in 2,500. In the northern hemisphere, the x coordinate of the central meridian is offset to have a value of 500,000 meters instead of zero, normally termed as "False Easting." The y coordinate is set to zero at the Equator. In the southern hemisphere, the False Easting is also 500,000 meters with a y offset of the Equator or False Northing equal to 10,000,000 meters. These offsets force all coordinates in the system to be positive.

In the Universal Military Grid System (UMGS), the polar areas, north of 84° N and south of 80° S, are projected to the Universal Polar Stereographic (UPS) Grid with the pole as the centre of projection and a scale factor 0.9994. They are termed "North Zone" and "South Zone."

Map projection also is dependent on the shape of the country. In the United States of America, the State Plane Coordinate System is established in which states with an east-west long axis, Tennessee, for example, use the Lambert Conformal Conic projection, whereas states with a north-south long axis, Illinois, for example, use the Transverse Mercator projection.. Not only a map projection and the map scale, but coordinate

measurement units are also an important part of any map. In order to be sure of the accuracy of data taken from a map, read carefully all information written along the border of the map and, if necessary, ask the National Mapping Agency for additional information.

A final plane coordinate system of relevance to geographic data modelling and analyses, particularly for satellite images and photographs, is an image coordinate system. A digital image system is not a right-handed Cartesian coordinate system since usually the initial point (0, 0) is assigned to the upper left corner of an image. The x coordinate, often called sample, increases to the right, but the y coordinate, called the line, increases down. Units commonly are expressed in picture elements or pixels. A pixel is a discrete unit of the Earth's surface, usually square with a defined size, often expressed in metres.

Often, in geodetic and cartographic practice, it is necessary to transform plane Cartesian two-dimensional coordinates to another plane two-dimensional coordinate system. The indirect method transforms plane two-dimensional coordinates into spherical or ellipsoidal coordinates by using so-called inverse map projection equations. Then, the method follows with appropriate map projection equations that give the result in the second plane, two-dimensional system. The direct method transforms plane coordinates from one system to another by using rotation, translation, scaling, or any other two-dimensional transformation. For more details, the reader should consult references.

9.5 Classes of Map Projections

Projections may be classified on the basis of geometry, shape, special properties, projection parameters, and

nomenclature. The geometric classification is based on the patterns of the network (the network of parallels of latitude and meridians of longitude). According to this classification, map projections are usually referred to as *cylindrical*, *conical*, and *azimuthal*, but there are also others. A complete description of these geometric patterns and associated names can be found in the references

An azimuthal projection also projects the image of the Earth on a plane. A map produced in cylindrical projection can be folded in a cylinder, while a map produced in conical projection can be folded into a cone. Firstly, let us accept that almost all map projections in use are derived by using mathematics, especially its part known as differential calculus. This process allows for the preservation of specific characteristics and minimizing distortion, such as angular relationships (shape) or area.

9.5.1 Cylindrical Projections

Cylindrical projections are those that provide the appearance of a rectangle. The rectangle can be seen as a developed cylindrical surface that can be rolled into a cylinder. Whereas these projections are created mathematically rather than from the cylinder, the final appearance may suggest a cylindrical construction. A cylindrical map projection can have one line or two lines of no scale distortion. Classic examples of cylindrical projections include the conformal Mercator and Lambert's original cylindrical equal area (Figure 9.5).

Cylindrical projections are often used for world maps with the latitude limited to a reasonable range of degrees south and north to avoid the great distortion of the polar areas by this projection method. The normal

aspect Mercator projection is used for nautical charts throughout the world, while its transverse aspect is regularly used for topographic maps and is the projection used for the UTM coordinate system described above.



a.



b.

Figure 9.5. The conformal cylindrical Mercator projection (a) and Lambert's cylindrical equal area projection (b).

9.5.2 Conical Projections

Conical projections give the appearance of a developed cone surface that can be furled into a cone. These projections are usually created mathematically and not by projecting onto a conical surface. A single line or two lines may exist as lines of no scale distortion.



a.



b.

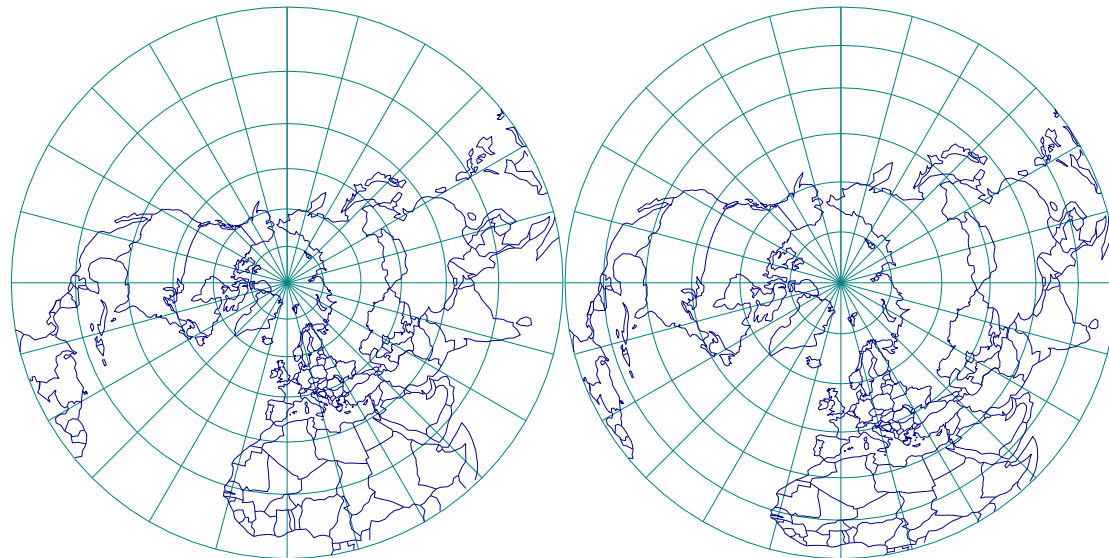
Figure 9.6. Lambert's conformal conic (a) and the Albers conical equal area (b) projections.

Classic examples of conical projections are Lambert's conformal conic and the Albers conical equal area

projection (Figure 9.6). Conical projections are inappropriate for maps of the entire Earth and work best in areas with a long axis in the east-west direction. This makes them ideal for representations of land masses in the northern hemisphere, such as the United States of America, Europe, or Russia.

9.5.3 Azimuthal Projections

Azimuthal projections are those preserving azimuths (*i.e.*, directions related to north in its normal aspect). A single point or a circle may exist with no scale distortion. Classic examples of azimuthal projections include the stereographic and Lambert's azimuthal equal area (Figure 9.7).



a.

b.

Figure 9.7. The stereographic (a) and Lambert's azimuthal equal-area (b) projections.

9.5.4 Other Classifications

Other classifications of map projections are based on the aspect (*i.e.*, the appearance and position of the graticule, poles or the equator in the projection). Aspect can be polar, equatorial, normal, transverse or oblique. Accordingly, there are polar projections, normal projections, equatorial projections, transverse projections and oblique map projections. These are names of individual sets of map projections and not a systematic categorization because, for example, a projection can be polar and normal at the same time. In theory, each projection can have any aspect. However, many projections are almost always used in certain aspects in order to express their characteristics as well as possible.

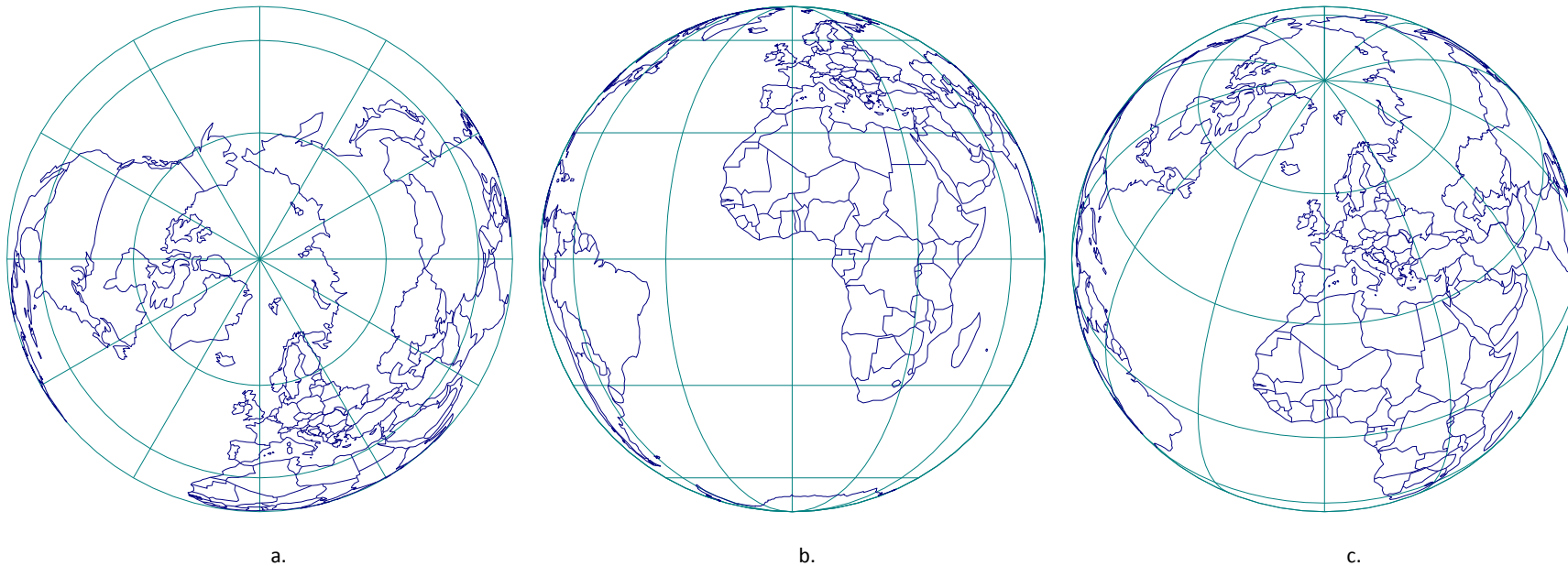


Figure 9.8. Orthographic projection in its normal (a), transverse (b) and oblique (c) aspects.

For example, many factors such as temperature, contamination breakout and biodiversity depend on the climate (*i.e.*, the latitude). For projections with a constant distance between parallels, the latitude in the equatorial aspect can be directly converted into vertical distance, facilitating comparison. Certain projections with graticules in normal aspect appearing as simple curves were originally defined by geometric constructions.

Considering most transverse and oblique projections have graticules consisting of complex curves, such projections were not systematically analysed prior to the computer era. In general, calculating oblique projections

for a particular ellipsoid is very complex and is not developed for all projections. Nevertheless, oblique projections have applications.

A map projection is a normal projection or it is in normal aspect if the appearance and position of the graticule, poles and the equator in the projection are the most natural and are usually determined by geometrical conditions. It is often determined by the simplest calculations or the simplest appearance of the graticule. The polar aspect is normal for azimuthal projections, while the equatorial aspect is normal for cylindrical projections. In azimuthal and conic projections, the graticule consists of straight lines and arcs of circles;

normal aspect cylindrical projections have graticules consisting only of straight lines forming a rectangular grid.

A map projection is a transverse projection or it is in transverse aspect if the appearance and position of the graticule, poles or the equator in the projection were derived by applying formulae for the normal aspect projection to a globe which was previously rotated by 90° around its centre, so that poles are in the equatorial plane.

A map projection is a polar projection or it is in polar aspect if the image of a pole is in the centre of the map.

It is often used as a synonym for normal aspect azimuthal projection.

A map projection is equatorial or it is in equatorial aspect if the image of the equator is in the centre of the map. The image of the Equator is placed in the direction of one of the main axes of the map, mostly horizontally. Equatorial projection often means normal aspect cylindrical projection.

A map projection is an oblique projection or it is in oblique aspect if it is neither polar nor equatorial, neither normal aspect nor transverse (Figure 9.8).

9.6 Preserving Specific Properties with Map Projections

Map projections are usually designed to preserve specific characteristics of the globe, such as areas, angles, distances, or specific properties such as great circles (intersections of the Earth and a plane which passes through the Earth's center) becoming straight lines. Maps with angles preserved are called *conformal* projections.

Maps with areas preserved are referred to as *equal-area* or *equivalent* projections

9.6.1 Preserving Angles

Gerardus Mercator in 1569 developed a cylindrical conformal projection that bears his name. He developed it to show loxodromes or rhumb lines, which are lines of constant bearing, as straight lines, making it possible to navigate a constant course based on drawing a rhumb line on the chart. The Mercator projection has meridians as equally spaced parallel lines, with parallels shown as

unequally spaced straight parallel lines, closest near the Equator and perpendicular to the meridians. The North and South Poles cannot be shown. Scale is true along the Equator or along two parallels equidistant from the Equator. Significant size distortion occurs in the higher latitudes and that is why the Mercator projection is not recommended for world maps (Figure 9.5a). The Mercator projection, a standard for marine charts, was defined for navigational charts and is best used for navigational purposes.

Transverse Mercator

The transverse Mercator, also known as a Gauss-Krüger projection, is a projection where the line of constant scale is along a meridian rather than the Equator. The central meridian and the Equator are straight lines. Other meridians and parallels are complex curves and are concave toward the central meridian. The projection has true scale along the central meridian or along two lines equidistant from and parallel to the central meridian. It is commonly used for large-scale, small area presentations. Due to the distribution of distortion, it is usually used by dividing the region to be mapped in three-degree or six-degree zones limited by meridians. This projection is widely used for topographic maps from 1:25,000 scale to 1:250,000 scale, and it is the basis of the UTM coordinate system.

Lambert Conformal Conic

The Lambert Conformal Conic (LCC) projection, presented by Johann Heinrich Lambert in 1772, shows meridians as equally spaced straight lines converging at one of the poles (Figure 9.6a). Angles between the meridians on the projection are smaller than the corresponding angles on the globe. Parallels are

unequally spaced concentric circular arcs centred on the pole, and spacing of the parallels increases away from the pole. The pole nearest the standard parallel is a point and the other pole cannot be shown. The scale is true along the standard parallel or along two standard parallels and is constant along any given parallel. The LCC projection is extensively used for large-scale mapping of regions with an elongated axis in the East-West directions and in mid-latitude regions. It is standard in many countries for maps at 1:500,000 scale, as well as for aeronautical charts of a similar scale.

Stereographic

The Stereographic projection, developed by the 2nd century B.C., is a perspective azimuthal projection that preserves angles (*i.e.*, is conformal). This projection is the only projection in which all circles from the globe are represented as circles in the plane of projection. The polar, Equatorial and oblique aspects result in different appearances of the graticule. The polar aspect is achieved by projecting from one pole to a plane tangent at the other pole. In this aspect, meridians are equally spaced straight lines intersecting at the pole with true angles between them. Parallels are unequally spaced circles centred on the pole represented as a point. Spacing of the parallels increases away from the pole. The Stereographic projection is used in the polar aspect for topographic maps of Polar Regions. The Universal Polar Stereographic (UPS) is the sister projection of the UTM for military mapping. This projection generally is chosen for regions that are roughly circular in shape. It is in use in oblique ellipsoidal form in a number of countries throughout the world, including Canada, Romania, Poland and the Netherlands. Different countries have different mathematical developments or versions of the Stereographic projection.

9.6.2 Preserving Areas

Lambert Cylindrical Equal Area.

The Cylindrical Equal Area projection was first presented by Johann Heinrich Lambert in 1772. It became the basis for many other similar equal area projections including the Gall Orthographic, Behrmann, and Trystan-Edwards projections. Lambert's original projection uses a single line of constant scale along the Equator (Figure 9.5b). Similar equal area projections are constructed using two parallels as the lines of constant scale. On the Lambert Cylindrical Equal Area projection, meridians are equally spaced straight parallel lines and the Equator is π times as long as the meridians. Lines of latitude are unequally spaced parallel lines furthest apart near the Equator and are perpendicular to the meridians. Changing the spacing of the parallels is the method used to preserve equal areas. Significant distance and angle distortion, however, results with the distortion greater in high latitudes near the poles. This projection is not often used directly for map construction, but it is a standard to describe map projection principles in textbooks and has also served as a prototype for other projections.

Mollweide

In 1805, Carl Brandan Mollweide developed a pseudocylindrical equal area projection on which the central meridian is a straight line one-half as long as the Equator forming an elliptical area of projection for the entire globe. The meridians 90° East and West of the central meridian form a circle on the Mollweide projection. Other meridians are equally spaced semiellipses intersecting at the poles and concave toward the central meridian. Parallels are unequally spaced straight lines and are perpendicular to the

central meridian. The parallels are farthest apart near the Equator with spacing changing gradually.



Figure 9.9. Logo of ICA in the Mollweide projection.

The North and South Poles are shown as points, and the scale is only true along latitudes $40^\circ 44'$ North and South and constant along any given latitude. The entire globe projected and centred on the Greenwich meridian is shown in Figure 9.9. The Mollweide projection has occasionally been used for world maps, particularly thematic maps where preservation of area is important. Different aspects of the Mollweide have been used for educational purposes, and it was chosen for the logo of ICA (Figure 9.9).

9.6.3 Compromise Projections

Map projections that are neither conformal nor equal area are called compromise projections. They are almost unlimited in variety. Among them are many important and useful projections.

Orthographic

The Orthographic projection, developed by the 2nd century B.C., is a perspective azimuthal projection that is neither conformal nor equal area. It is used in polar, Equatorial and oblique aspects and results in a view of

an entire hemisphere. The polar aspect of the projection has meridians that are straight lines and intersect the central pole with the angles between meridians being true. The pole is a point and the parallels are unequally spaced circles centred on the pole. The spacing parallels decrease away from the pole. Scale is true at the centre and along the circumference of any circle with its centre at the projection centre. The projection has a globe-like look (Figure 9.8), and is essentially a perspective projection of the globe onto a plane from an infinite distance (orthogonally). It is commonly used for pictorial views of the Earth as if seen from space.

Gnomonic

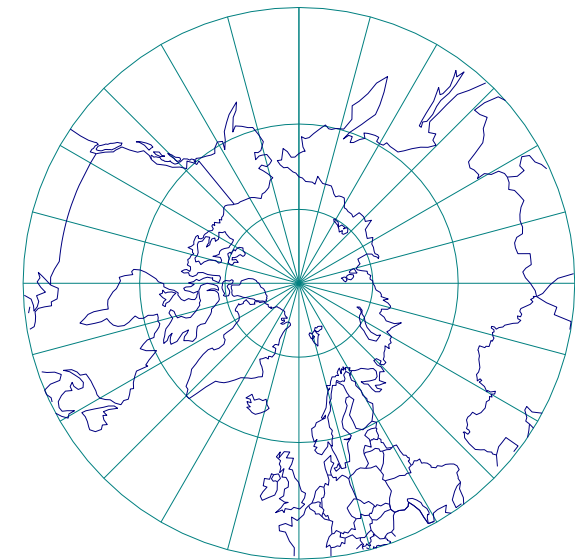


Figure 9.10. The Gnomonic projection, which maps great circles to straight lines.

The Gnomonic projection is neither conformal nor equal area. It is a perspective azimuthal projection with the point of projection at the centre of the Earth, which is the source of the name (*i.e.*, the centre of the Earth where the mythical gnomes live). It was developed by the Greek Thales, possibly around 580 B.C. All great circles on the projection, including all meridians and the Equator, are shown as straight lines, a property unique to this projection (Figure 9.10).

The graticule appearance changes with the aspect, as with other azimuthal projections. Meridians are equally spaced straight lines intersecting at the pole with true angles between them in the polar aspect. Parallels are unequally spaced circles centred on the pole as a point, and the spacing of the parallels increases from the pole. The projection only can show less than a hemisphere. Scale increases rapidly with distance from the centre. Its usage results from the special feature of representing great circles as straight lines, and it thus assists navigators and aviators in determining the shortest courses.

Azimuthal Equidistant

In this polar aspect projection, meridians are equally spaced straight lines intersecting at the central pole. Angles between them are the true angles. Parallels are equally spaced circles, centred at the pole, which is a point. The entire Earth can be shown, but the opposite pole is a bounding circle having a radius twice that of the Equator. In its equatorial aspect, meridians are complex curves, equally spaced along the Equator and intersecting at each pole. Parallels are complex curves concave toward the nearest pole and equally spaced along the central meridian and the meridian 90° from the central meridian. The scale is true along any straight

line radiating from the centre of projection. It increases in a direction perpendicular to the radius as the distance from the centre increases. Distortion is moderate for one hemisphere but becomes extreme for a map of the entire Earth. The distance between any two points on a straight line passing through the centre of projection is shown at true scale; this feature is especially useful if one point is the centre.

This projection is commonly used in the polar aspect for maps of Polar Regions, the Northern and Southern Hemispheres, and the "aviation-age" Earth. The oblique aspect is frequently used for world maps centred on important cities and occasionally for maps of continents. The Azimuthal Equidistant projection was recognized by the UN and used on the UN's flag (Figure 9.11).



Figure 9.11. The azimuthal equidistant projection for preserving distances on the UN's flag.

Winkel Tripel

The Winkel Tripel projection is neither conformal nor equal area. It was presented by Oswald Winkel of Germany in 1921.



Figure 9.12. Winkel Tripel projection.

The projection was obtained by averaging coordinates of the Equidistant Cylindrical and Aitoff projections. Winkel applied the name "Tripel," normally meaning triple, because the Aitoff projection is an equatorial aspect of one hemisphere of the Azimuthal Equidistant projection, on which horizontal coordinates have been doubled and meridians have been given twice their original longitudes.

The central meridian is straight. Other meridians are curved, equally spaced along the Equator and concave toward the central meridian.

The Equator and the poles are straight. Other parallels are curved, equally spaced along the central meridian and concave toward the nearest pole. Poles are straight lines about 0.4 as long as the Equator, depending on the latitude of the standard parallels. Scale is true along the central meridian and constant along the Equator.

Distortion is moderate except near outer meridians in Polar Regions. The Winkel Tripel is used for whole-world maps (Figure 9.12).

9.7 Modern Approaches to Map Projections

9.7.1 Web Mercator

Many major online street mapping services (Bing Maps, OpenStreetMap, Google Maps, MapQuest, Yahoo Maps, and others) use a variant of the Mercator projection for their map images. Despite its obvious scale variation at small scales, the projection is well suited as an interactive world map that can be zoomed into seamlessly to large-scale (local) maps, where there is relatively little distortion due to the variant projection's near-conformality.

The scale factor at a point on a conformal map projection (such as the spherical Mercator or the ellipsoidal Mercator) is uniform in all directions. This is not true on a Web Mercator. Let us denote with m the scale factor in the N/S meridian direction and with n the scale factor in the E/W parallel direction. Then $m = n$ because the scale factor at a point is the same in all directions on the spherical Mercator projection. In other words, the spherical Mercator is conformal.

The equations for the ellipsoidal Mercator are a little more complicated, especially in Northing. The parameters a (semi-major axis) and e (eccentricity) are given for the selected ellipsoid. Again $m = n$ because the scale factor at a point is the same in all directions on the ellipsoidal Mercator projection. In other words, the ellipsoidal Mercator is conformal.

Web Mercator is the mapping of WGS84 datum (*i.e.*, ellipsoidal) latitude/longitude into Easting/Northing using spherical Mercator equations (where $R = a$). This projection was popularized by Google in Google Maps (not Google Earth). The reference ellipsoid is always

WGS84, and the spherical radius R is equal to the semi-major axis of the WGS84 ellipsoid a . That's "Web Mercator."

The scale factor at a point is now different for every direction. It is a function of the radii of curvature in the meridian and the prime vertical and the direction α . For the Web Mercator, m and n are not equal. Thus, the Web Mercator is not a conformal projection.

If somebody uses the Web Mercator for printing out directions to a new restaurant across town or for visualization on his/her computer screen or for other purposes on the web, there will be no problem. But the Web Mercator is a projection that has jumped from one domain of use (the web) to another domain of use (GIS) where it is leading another life. Witnesses are the EPSG, Esri and FME codes for the Web Mercator. Surveyors and GIS professionals need to know that the Web Mercator is not conformal. If distance computations on the Web Mercator are done simply (as they can be done on a conformal projection), they will be wrong. If done correctly, they will be laborious.

For an area the size of the NW quadrisphere (North America), the differences appear slight. It turns out that the Eastings are identical. The differences are in the Northings. There is no Northing difference at the Equator, but by 70 degrees North, the difference is 40 km. This NS stretching in the Web Mercator is the reason for its non-conformality.

Mercator projections are useful for navigation because rhumb lines are straight. These are lines of constant true heading that navigators used to sail before GPS. So, we have to have in mind that straight lines on a Web Mercator are not rhumb lines.

To summarize about the Web Mercator:

- The Web Mercator is cylindrical;
- Its meridians are equally spaced straight lines;
- Its parallels are unequally spaced straight lines but in a different way than a conformal Mercator;
- Its loxodromes (rhumb lines) are not straight lines;
- It is not perspective;
- Its poles are at infinity and
- It was not presented by Mercator in 1569, but by Google recently.
- It is not conformal.

9.7.2 Map Projection Transitions

Map Projection Transitions is an example of multiple applications offered by Jason Davies. The web page (<http://www.jasondavies.com/maps/transition>) presents a world map with graticule and country borders in the oblique Aitoff projection with the South Pole. The map is not static, but animated. The South Pole moves toward the bottom and Earth rotates around its poles. The animation lasts five seconds, after which the projection changes and movement continues for five seconds, after which the projection changes again. Names of projections appear in a separate window. There are a total of 56 projections. The South Pole eventually becomes invisible and the North Pole appears at the top. Various parts of Earth appear in the centre of the map by rotating around the poles (Figure 9.13).

By clicking *Pause*, animation stops and it is possible to select another projection. By left-clicking, it is possible to

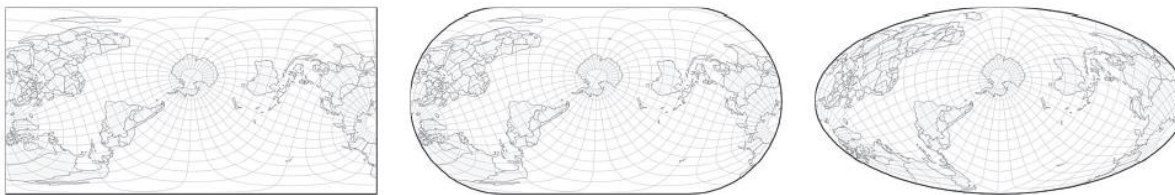


Figure 9.13. From the Map Projection Transitions application (<http://www.jasondavies.com/maps/transition>)

move the picture around and select projection aspect—normal, transversal or any of numerous oblique projections. Differences between two projections can be seen clearly in such a way. For example, one is able to select the Ginzburg VI projection and its normal aspect by moving the mouse. If one wants to see how that projection's graticule is different from the similar Winkel Tripel projection, it can be done by clicking on the *Winkel Tripel* projection on the drop-down menu. The picture on the screen is going to change to the Winkel projection and differences are going to be clear.

If one clicks on *Maps*, there is a series of new interesting applications about interrupted maps, butterfly-shaped maps, retro-azimuthal projections and other projections. It is possible to use the mouse to move pictures in many of those applications. For example, by selecting the interrupted sinusoidal projection, a world map in three segments is going to appear. The mouse can be used to move parts of Earth from one segment to another, and the slider at the bottom of the screen can be used to change the number of segments from an uninterrupted world map to a representation in 24 segments.

A similar option is available for the Berghaus (Snyder and Voxland, 1989) star projection. The application *Azimuth*

and *Distance from London* enables using the mouse to obtain distances and azimuths from London to any point on Earth in world maps in oblique equidistant cylindrical and oblique equidistant azimuthal projection. If an application's accompanying text mentions a projection, there is a link to Wikipedia where there is detailed information on the projection.

9.7.3 Research on New Map Projections

In 2007, inspired by Robinson's method, B. Jenny, T. Patterson and L. Hurni produced the Flex Projector interactive program, which enables the user to create new world map projections with ease. It supports the normal aspect of cylindrical projections. The program is free and open source and works under Linux, Mac OS X and Windows. By executing the program, a world map in the Robinson projection appears on the screen (Figure 9.14). The right side of the screen includes sliders for changing lengths of parallels. Clicking the Distance button brings up sliders for changing distances of parallels from the Equator. Parallel curvatures (Bending) and distances between meridians (Meridians) can also be changed. The Linked Sliders option enables the user to move each slider separately or several at once. The next option Move is used to choose the shape of the

curve along which the sliders are moved. The ratio between the central meridian and the Equator can be changed with the Proportions (Height/Width) slider. Instead of modifying the Robinson projection, one can start from any of a number of provided projections from the three groups mentioned. If the result is unsatisfying, one can use the option Reset Projection to go back to the initial projection. The option can be found in the upper right corner of the screen.

Clicking Display opens up additional options. The length of the central meridian can be changed, graticule density can be chosen, distortion ellipses can be drawn in nodes of the graticule, and area and maximum angle distortion isograms can be drawn. The background of newly created projection can include the graticule and continent outlines in any activated projection (Show Second Projection). The bottom left corner of the screen includes numerical indicators of the summary length, area and angle distortions for all activated projections and the projection just created (Figure 9.14).

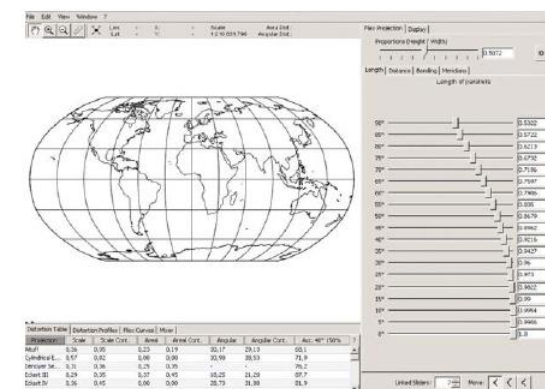


Figure 9.14. Interface of the Flex Projector program.

Flex Projector can import and export vector and raster data in several formats. The program is recommended to everyone wanting to try creating a new world map projection, and it can also be applied in teaching map projections.

The techniques for combining two source projections to create a new projection allow for the creation of a large variety of projections. The mentioned techniques can also be extended. For example, the Geocart software by Mapmathematics can blend projection parameters, such as the latitude of standard parallels, between two source projections. Alternatively, more than two projections can be combined to form a new one. The extreme case would be an infinite number of differently parameterized projections, which is the concept behind polyconic and poly-cylindric projections. There are alternative methods for creating a new projection from scratch, deriving it from existing ones or adjusting projection parameters to create a new one. Some of these techniques are used in the adaptive composite projections for web maps, a new field of map projection research. The goal of this research is to develop an alternative to the Web Mercator projection for small-scale web maps, where maps automatically use an optimum projection depending on the map scale, the map's height-to-width ratio, and the central latitude of the displayed area.

9.8 Suggested Projections

The reason we have so many map projections is because none serves every need. The selection of an appropriate map projection for a given application depends on a variety of factors, including the purpose of the map, the type of data to be projected, the region of the world to be projected and scale of the final map. Advice on

selection is available from a variety of print and web sources (see Further Reading section). In GIS, large-scale datasets (small area extent) commonly are projected with a conformal projection to preserve angles. For such applications, area distortion is so small over the geographic extent that it is negligible and an area preserving projection is not needed. Commonly, large-scale data files are used in GIS applications of limited geographic extent (*e.g.*, a watershed, a county or a state). The two most commonly used projections for these scales are the Lambert Conformal Conic and the Transverse Mercator, which are the basis of the UTM and most of the USA State Plane coordinate systems. For general-purpose world maps, our recommendation is not using any cylindrical map projection but some of pseudo-cylindrical (*e.g.*, Robinson or a compromise projection like the Winkel Tripel).

9.9 Conclusions

Map projections and coordinate transformations are the basis of achieving a common frame of reference for geographic information. The requirement of a common ellipsoid, datum, map projection, and finally plane coordinate systems make it possible to use plane geometry for all types of spatial overlay and analysis. Projection of geographic data from the ellipsoidal Earth to a plane coordinate system always results in distortion in area, shape, distance, and other properties. With appropriate selection of a projection, the user can preserve desired characteristics at the expense of others. In this chapter we have briefly examined basic concepts of the basis of coordinate systems and map projections. For a more in-depth treatment, the reader is referred to the texts and sources referenced in the Further Reading section.

9.10 Further Reading

Further references and an exercise with questions and answers are given in Chapter 18.

Google e-Books on Map Projections

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